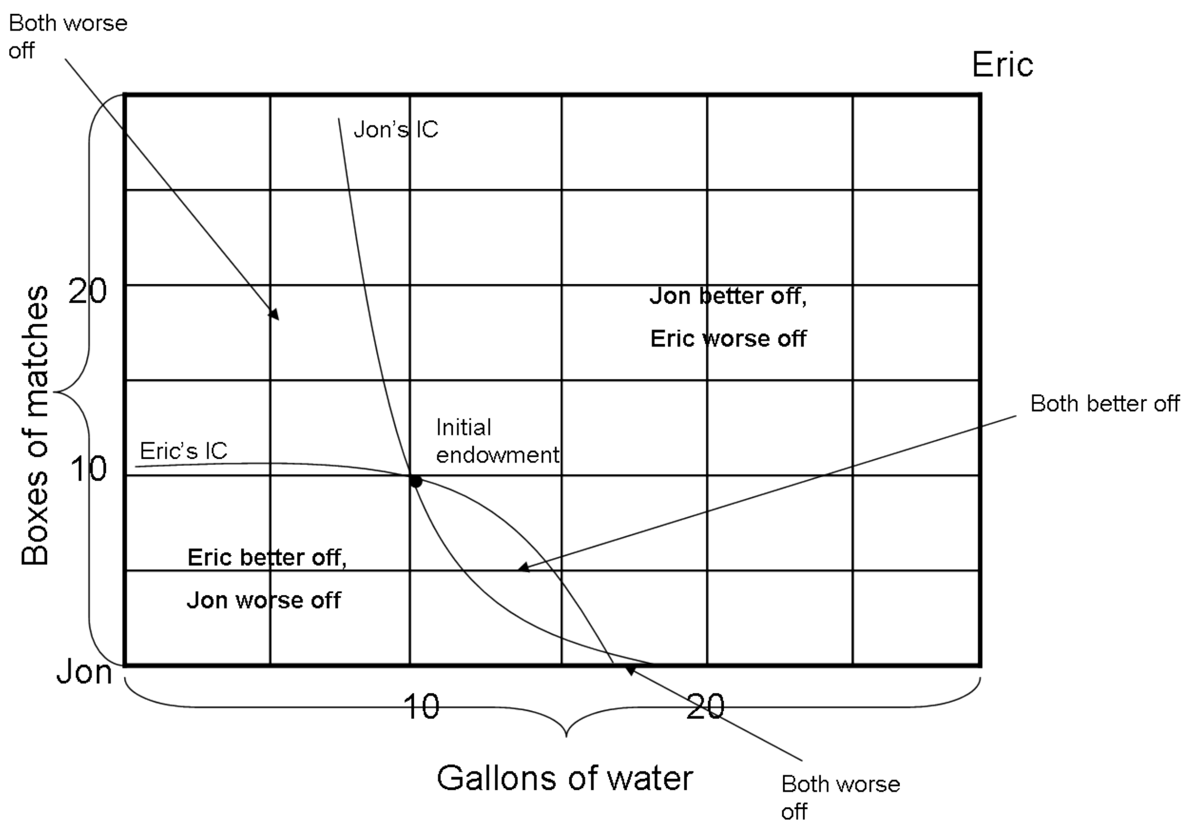
EEP 100 - Problem Set 2
Date Due: Thursday 10/08/2009

1. Edgeworth box: Suppose Jon and Eric are stranded on a deserted island. Jon has with him 10 boxes of matches and 10 gallons of drinking water. Eric has 20 boxes of matches and 20 gallons of water. At the initial allocation, Jon's $MRS_{water,matches} = 4$ (i.e. Jon would be just as well off if he gave away 4 boxes of matches in return for 1 gallon of water). Eric's $MRS_{water,matches} = 0.5$.

   (a) Using Graph (1a) on the last page, label the origins, axes, and the initial endowment. Using the given MRS's, draw and label Eric and Jon's indifference curves (use generically shaped, convex, indifference curves).



For the above graph, the only really important information is given the relative slopes at the initial endowments. You could have reversed Jon and Eric's origins or put water on the vertical axis instead.

From the MRS, we know that Jon would trade 4 boxes for matches for 1 gallon of water, giving him a slope of -4 matches/water. For Eric, his slope is given by -0.5 matches/water. Thus, if the vertical axis contained matches, Jon's slope should be steeper than Eric's slope at the initial endowment point. Alternatively, if water was on the vertical axis, then Jon's slope would be -0.25 water/matches and Eric's slope would be given by -2 water/matches. Therefore, when water is on the vertical axis, Eric's slope should be steeper than Jon's. These cases are true independently of which corners represent Jon and Eric's origins.

(b) Give one trade that makes both Jon and Eric better off. Identify the region of the Edgeworth box that represents where Jon and Eric are better off.

Because Jon would trade more matches for water compared to Eric, we know that Jon values matches relatively less and water relatively more. Therefore, we know that for a trade to make them both better off, Jon must give matches and receive water. One such trade is: Jon gives 3 boxes of matches in return for 1 gallon of water. Since Jon would be just as happy to give 4 boxes for one gallon, he is better off. Since Eric only needs to receive half a match for one gallon, he is also better off. Any ratio of matches/water trading within (0.5, 4) would make both better off. The regions for parts (b), (c), and (d) are identified on the figure.

(c) Give one trade that makes Jon better off and Eric worse off. Identify the region of the Edgeworth box that represents where Jon is better off and Eric is worse off.

One such trade is given by Jon giving 0.25 boxes of matches for one gallon of water. We know that Jon would be willing to give 4 boxes of matches so he is better off. We know that Eric needs to receive at least 0.5 boxes of matches so he is worse off. Any such trading ratio matches/water between (0,0.5) would satisfy this requirement. (For a trade, both people need to give and receive positive quantities).

Alternatively, Jon could give 1 gallon of water for 5 boxes of matches. Since Jon only needs to receive 4 boxes of matches to be just as happy, he is better off. Since Eric needs to receive 10 gallons of water to be just as happy, he is worse off. Any such trading ratio of matches/water between $(4, infty)$ would satisfy this requirement.

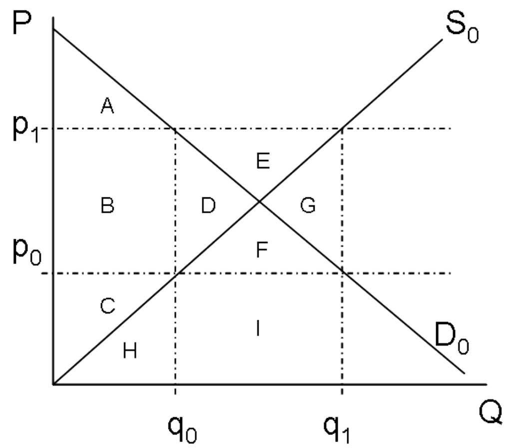
(d) Give one trade that makes both Jon and Eric worse off. Identify the region(s) of the Edgeworth box that represents where Jon and Eric are worse off.

Again, because Jon would trade more matches for water compared to Eric, we know that Jon values matches relatively less and water relatively more. Therefore, for a trade to make them both worse off, Jon must give water and receive matches. One such trade is: Jon gives 1 gallon of water for 3 boxes of matches. Since Jon needs to receive 4 boxes for 1 gallon, he is worse off. Since Eric needs to receive 6 gallons of water for 3 boxes of matches, he is also worse off. Any ratio of water/matches trading within (0.5, 4) would make both better off.

2. Agricultural policy and deadweight loss: Consider the following two agricultural policies used by the U.S. government to support farmers.

   Loan rate: (Despite the name of the program, it has very little to do with loans.) Under the loan rate program, the government guaranteed farmers a price (p_1 on the graph below) for their goods. Under that guarantee, the farmers would decide how much to produce and they would sell their produce on the market at that guaranteed price. The government would buy up any surplus produce and pay the farmers (p_1 per unit) for that produce.

   Target price: Under the target price program, the government again guaranteed farmers p_1 for their goods and farmers would decide how much to produce according to that price. This time, though, farmers would sell **all** their goods on the market and receive the market-clearing price (the price that makes supply and demand equal) at that quantity. If the market-clearing price was below the target price of p_1, the government would pay the farmers the difference for every unit sold.



(a) Identify the price received by farmers, the quantity produced by farmers, the price paid by consumers, the quantity purchased by consumers, and the quantity purchased by the government under each policy by filling in the Table (2a) on the last page.

Table (2a)

	Loan rate	Price support
Price received by farmers	p_1	p_1
Quantity produced by farmers	q_1	q_1
Price paid by consumers	p_1	p_0
Quantity bought by consumers	q_0	q_1
Quantity bought by government	$q_1 - q_0$	0

Under both policies, the government is guaranteeing the higher price, so producers receive p_1 and produce q_1 according to their supply curve. Under the loan rate, the producers sell at p_1 because they know that all of the quantity will be bought. Thus, the consumers pay p_1 and buy q_0 from their demand curve and the government is left buying all the surplus $q_1 - q_0$. Under the target price (price support), to clear the market (make supply equal to demand), consumers must pay p_0 and buy $q1$. Since consumers buy all the units, the government does not buy any units.

(b) Using the labeled areas in the above graph, complete the first column of Table (2b) on the last page associated with the loan rate.

(c) Using the labeled areas in the above graph, complete the second column of Table (2b) associated with the price support.

	Loan rate	Price support
Consumer surplus	A	A+B+D+F
Producer surplus	B+C+D+E	B+C+D+E
Government expenditure	-D-E-G-F-I	-B-D-E-F-G
Total welfare	A+B+C-G-F-I	A+B+C+D-G
DWL	D+G+F+I	G

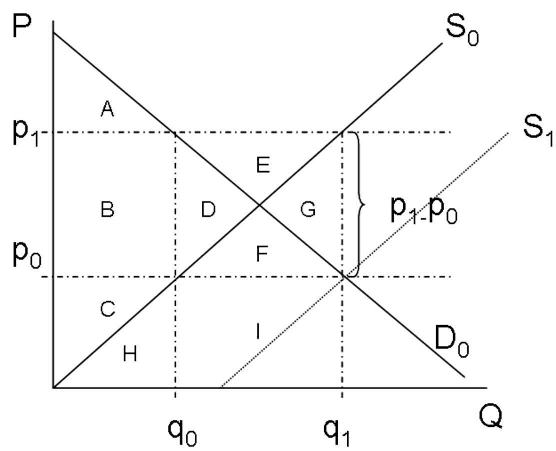
Recall that consumer surplus is given by the area under demand and above consumer price from zero to quantity bought. For the loan rate, consumers pay p_1 and buy q_0 so their surplus is A. For the price support, they pay p_0 and buy q_1 so their surplus is given by $A + B + D + F$.

Producer surplus is given by the area above supply and below produce price. For both the loan rate and the price surplus, producers receive p_1 and produce q_1 so their surplus is area $B + C + D + E$ for both policies.

Government expenditure is given by the price the government times the number of units they pay for. For the loan rate, the government pays the full price p_1 for units $q_1 - q_0$. This expenditure is represented by area $D + E + F + G + I$. For the price support, the government pays the difference in the target and market price $p_1 - p_0$ for all the units q_1 (even though they don't actually buy the units). Thus the area is given by $B + D + E + F + G$. Since they government puts out money, the welfare contribution is negative.

Total welfare is given by the sum of the top three columns, accounting for the fact that government expenditure is negative. Deadweight loss is found by comparing the total welfare under no policy with total welfare under the policy. Under no policy, we have total welfare as area $A + B + C + D$. Comparing this to the loan rate, we have $DWL = welfare_{nopolicy} - welfare_{loanrate} = A + B + C + D - (A + B + C - F - G - I) = D + F + G + I$. (You could have given it as a negative quantity, to convey that it is a loss.) For the price support, we have $DWL = welfare_{nopolicy} - welfare_{pricesupport} = A + B + C + D - (A + B + C + D - G) = G$. The price support has a much smaller deadweight loss.

(d) The price support program is equivalent to a government subsidy. Using the above graph, identify the size of the subsidy that would produce identical outcomes to the price support program.



The identical subsidy would need to shift the supply out so that the new equilibrium occured at (q_1, p_1). The size of the subsidy is given by the vertical shift, $p_1 - p_0$.

Note: The U.S. government has been using loan rate type policies to support farmers since 1929. In the 1996 and 2002 farm bills, the government implemented a target price approach (sometimes in combination with the loan rate). In addition to supporting domestic producers, these policies have led to overproduction of some goods (relative to the unregulated market). For example, the introduction of inexpensive corn syrup into many processed food products, which may be associated with the rising rate of obesity, is one of the by-products of a large government surplus of corn. See Modern Industrial Organization (Carlton and Perloff) p.718-721 for more information.

3. Production

A firm uses labor to produce widgets with a production function of $Q(L) = L^{\frac{1}{2}}$.

Given that the price of labor is \$2/unit and the firm's fixed cost is \$8, give a graphic representation of:

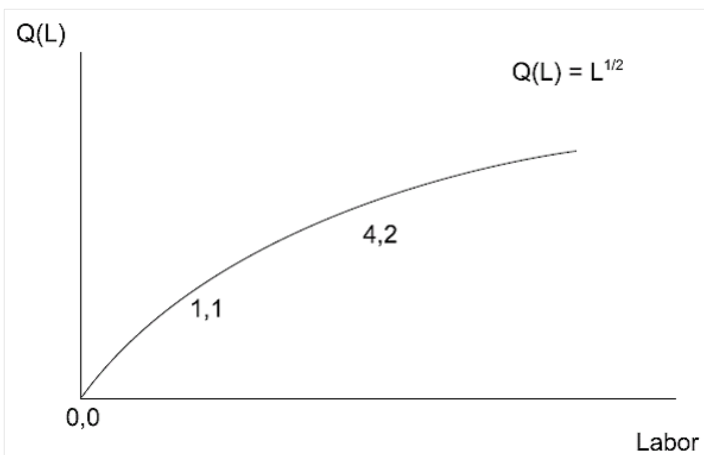
In starting to think about the problem, we set up the equation for total costs.

$$TC = FC + VC = 8 + 2L$$

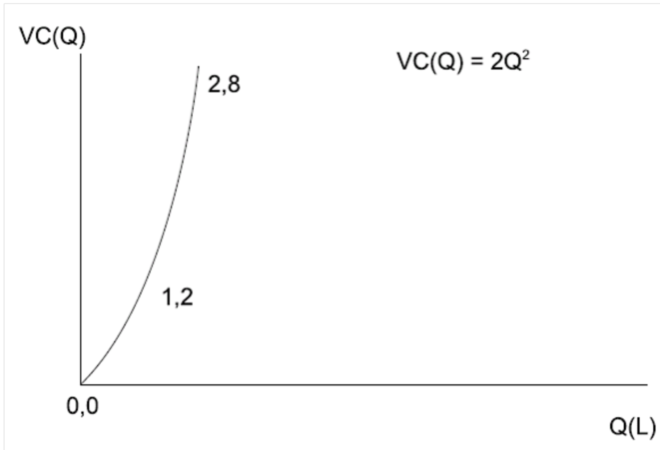
From the production function, we know that $L = Q^2$. Plugging in, we have

$$TC = 8 + 2Q^2$$

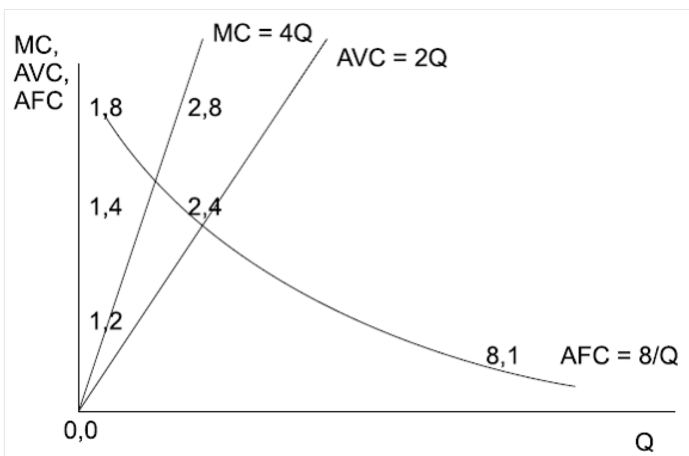
(a) The production function
The production function gives output as a function of inputs. The equation was already given.



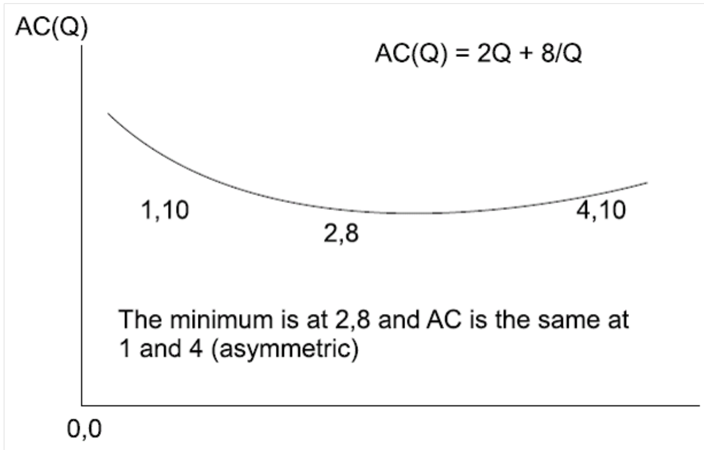
(b) The variable cost function
Variable costs are the portion of total costs that vary with Q.



(c) The marginal cost, average variable cost and average fixed cost functions
Marginal costs are given by the derivative of total costs. Average variable costs are given by variable costs divided by Q. Average fixed costs are given by fixed costs divided by Q.



(d) The average cost function
Average costs refers to average total costs and are thus given by total costs divided by Q.



For full credit, label the axes, write the algebraic form of the function next to its curve, and find *at least* three points on each curve. (Hint: *always* label the point where the curve touches an axis.) If the curve has a minimum or maximum (at a point *other* than zero or infinity), label that point. Show your work.

4. Price-taker Profits

   Assume that this firm is a price-taker. Find:

   (a) *The firm's profit function, profit-maximizing quantity (Q^*), and optimized profit function (the one that includes the price, P).*

   Note: I should have used q_i instead of Q to make it clear that this was one firm among many. Sorry.

$$\pi(q_i) = q_i P - 2q_i^2 - 8$$

$$\frac{d\pi_i}{dq_i} = P - 4q_i \overset{set}{=} 0 \Rightarrow q_i^* = \frac{P}{4} \Rightarrow \pi^* = \frac{P^2}{8}.$$

   Since FC is not a decision variable (it's not endogenous), it's not included.

   (b) *Given a widget price of \$4, find the quantity produced if the firm is already in business (short-run decision). Find the quantity produced if the firm is deciding whether to start up.*

   The short-run decision ignores fixed costs. Given $P = 4$, $q_i^* = 1$.

Ignoring FC, profit from this decision is $\pi_i = (1)(4) - 2(1)^2 = 2$. If we include FC, profit would be -6, which indicates that a firm deciding to start up should not enter the market, i.e., it should produce $q_i^* = 0$.

(c) *Find the short run and start up quantity produced if widgets sell for \$12.*

Same as 3(b) but with a higher price.

The short-run decision ignores fixed costs. Given $P = 12$, $q_i^* = 3$.

Ignoring FC, profit from this decision is $\pi_i = (3)(12) - 2(3)^2 = 18$. If we include FC, profit would be 10, so a start up *will* enter the market.

(d) *In the perfectly-competitive long run, what's Q^* and P^*?*

In the long run, fixed costs are included in production decisions (endogenous) and price falls until profits are zero. Thus,

$$pq_i - 2q_i^2 - 8 \overset{set}{=} 0 \Rightarrow p^* = 8, q_i^* = 2.$$

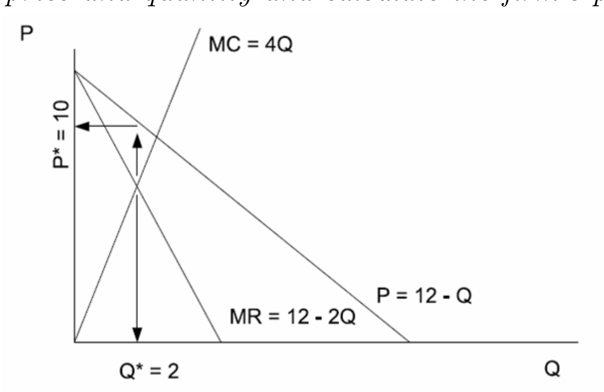
5. Now assume that the firm is a monopolist facing a demand function of $Q = 12 - P$. Find:

(a) *The firm's profit function and profit-maximizing quantity (Q^*).*

$$\pi(Q) = QP - 2Q^2 - 8 \Rightarrow \pi(Q) = Q(12 - Q) - 2Q^2 - 8 \Rightarrow 12Q - 3Q^2 - 8$$

$$\frac{\mathrm{d}\pi}{\mathrm{d}Q} = 12 - 6Q \overset{set}{=} 0 \Rightarrow Q^* = 2, P^* = 10 \Rightarrow \pi^* = 4.$$

(b) *Draw curves for demand, supply and marginal revenue. Label equilibrium price and quantity and calculate the firm's profits.*



(c) *Is it possible for a firm that controls the market to produce zero? Explain.*

Yes. Either because marginal cost is greater than any price on the demand function (e.g., MC for the first unit is 13...) or because fixed costs are greater than the profit maximizing quantity, i.e., a firm that controls the market will make a profit of zero from zero production and profit of less than zero for any production quantity.