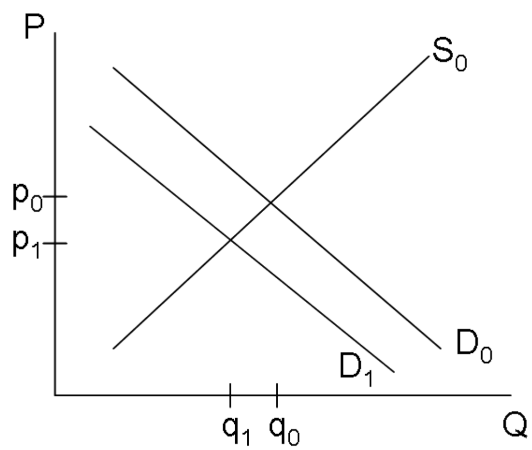
EEP 100 - Problem Set 1
Solutions

1. Suppose the U.S. government decides to expand New York city's ad campaign (http://www.nyc.gov/html/doh/html/pr2009/pr057-09.shtml) against drinking soda across the entire country. Assuming the campaign is effective, examine the effects of the ad campaign on the market for soft drinks:
a)Using a generic downward sloping demand curve and a generic upward sloping supply curve, graph the original supply and demand curves as well as the curves after the change. Label the curves as well as the original and final equilibrium prices and quantitites.

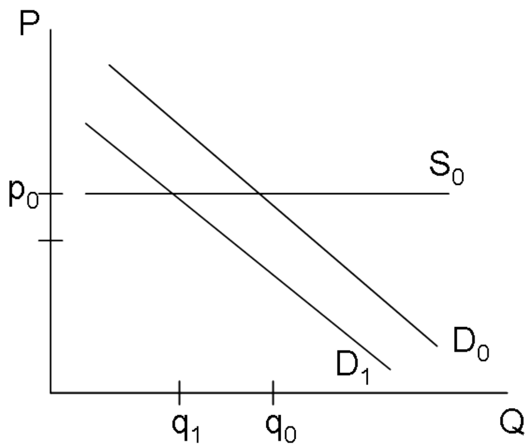
Solution:



b) State how supply and demand change. Do the curves shift in or out, or do the equilibrium points move up or down the curves? State how the equilibrium prices and quantities change (increase or decrease).

Solution: The ad campaign affects consumer preferences so that at any given price, consumers will consume less quantity than before. This results in a demand shift inward from D_0 to D_1. Supply does not shift since preferences do not affect any of the technology or costs of producers, but the equilibrium moves down the supply curve. The equilibrium quantity and prices both decrease from q_0 to q_1 and p_0 to p_1, respectively.

c) Repeat parts a) and b) with the assumption that supply is perfectly elastic. Do the equilibrium price and quantity change more or less when supply is perfectly elastic?



Solution:
Given the same inward shift as below, demand shifts from D_0 to D_1. Supply does not shift. Under a perfectly elastic supply curve, the price does not change at all (changes less than before), and the equilibrium quantity decreases more because the horizontal distance from D_0 to D_1 is greater than the diagonal distance

d) Who are the winners and losers of this policy on the supply side and on the demand side? Name one group in each category (winners-supply, losers-supply, winners-demand, losers-demand).

There are many possible answers for this question, but you needed to recognize that the supply side refers to producers and the demand side refers to consumers. Retailers are usually included in the supply side and the government is a third party (neither supply nor demand side).

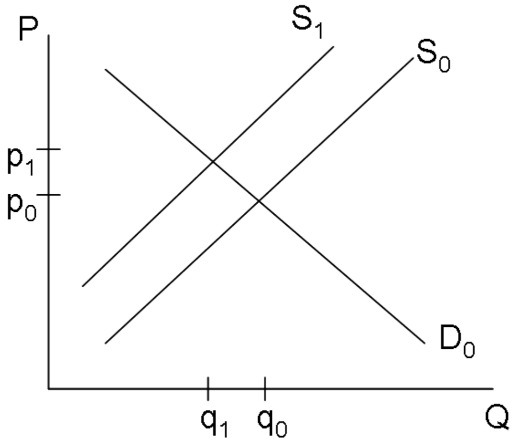
Possible solutions:

- Supply side winners: producers of soda substitutes (like juice)
- Supply side losers: soda producers (since they sell less at a lower price)
- Demand side winners: people who continue to drink soda (since they pay less)
- Demand side losers: people who drink soda substitutes face higher relative price; soda drinkers can also be considered losers in the long run if you consider health effects.

2. Suppose the FDA decides to put stricter standards on the fertilizer that can be used in the production of organic apples so that the cost of fertilizer rises. Analyze the effects on the organic apple market:

a) Repeat part a) from question 1.

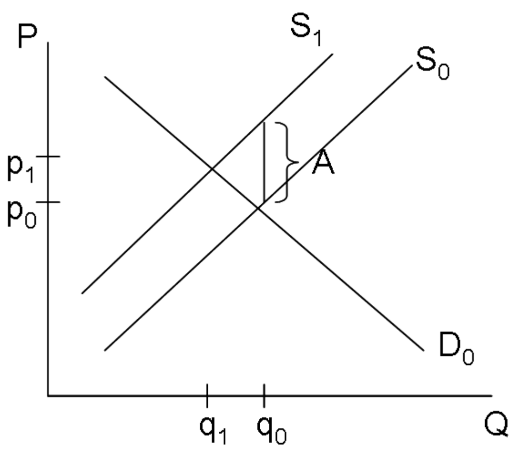
Solution:

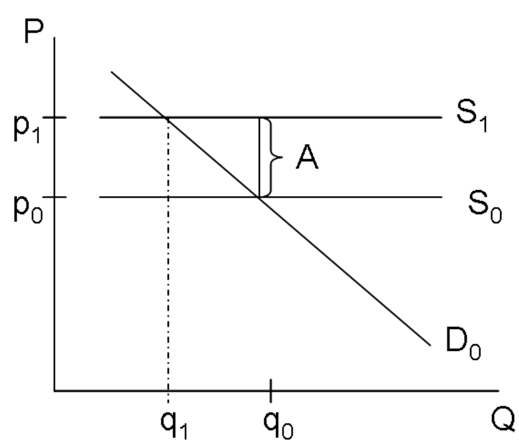


b) Repeat part b) from question 1.

Since fertilizer is an input to growing apples, the rising cost of fertilizer will result in a upward (inward) shift in supply. That is, for any given quantity, it now costs more to produce. The demand curve does not shift but quantity demanded moves up the demand curve. The equilibrium price is now higher and the equilibrium quantity has fallen.

c) Repeat parts a) and b) with the assumption that supply i smore elastic than you had previous assumed. Do the equilibrium price and quantity change more or less when supply is more elastic (given the same cost change for fertilizer)?





The same cost change for fertilizer means that the vertical shift in supply is the same (denoted as A in the figure). To see what happens when supply is more elastic, you can plot the most extreme case of perfect elasticity. From the figure, since supply is now flat, there is a larger change in price (the new equilibrium price is higher than before). This means that the equilibrium has shifted further up the demand curve, and the change in equilibrium quantity is also larger (the new equilibrium quantity is lower than before).

d) Repeat part d) from question 1.

Possible solutions:

- Supply side winners: high quality fertilizer producers (since organic farmers are forced to buy from them), non-organic apple farmers (since consumers will substitute away from organic apples due to the higher prices)
- Supply side losers: organic apple farmers (sell less)
- Demand side winners: organic apple consumers (since they get higher quality produce); fish consumers (since there is less toxic run-off from lower quality fertilizer), consumers of organic apple substitutes (since they face a lower relative price)
- Demand side losers: organic apple consumers (since they face a higher relative price)

3. Consider an economy with a representative consumer and two goods, 1 and 2. Assume that the consumer's preference can be represented by a Cobb-Douglas utility function: $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, where x_1 is the demand for good 1 and x_2 is the demand for good 2, α, β are both positive real numbers and $\alpha + \beta = 1$. Suppose that the price of good 1 is p_1, the price of good 2 is p_2, and that the consumer has an income m. Assume that p_1, p_2, m are all positive. Derive the demand function for each good, and show that the share of income spent on good 1 is α and the share spent on good 2 is β.

Solution: From the question we can write the representative consumer's utility maximization problem as:

$$\underset{x_1, x_2 \geq 0}{Max}\, U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

$$s.t.\ p_1 x_1 + p_2 x_2 \leq m$$

Since the utility function is strictly concave in (x_1, x_2), then we can use the Lagrange multiplier method.

The lagrangian can be written as:

$$\mathcal{L}(x_1,\, x_2,\, \lambda) = x_1^{\alpha} x_2^{\beta} + \lambda\,(m - p_1 x_1 - p_2 x_2)$$

$$\implies F.O.C.s: \begin{cases} \frac{\partial \mathcal{L}}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{\beta} - \lambda p_1 = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta-1} - \lambda p_2 = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0 \end{cases} \implies \begin{cases} x_1^\star = \frac{\alpha m}{p_1} \\ x_2^\star = \frac{\beta m}{p_2} \end{cases}$$

Therefore the demand functions for both goods are:

$$\begin{cases} x_1^\star = \frac{\alpha m}{p_1} \\ x_2^\star = \frac{\beta m}{p_2} \end{cases}$$

Since $p_1 x_1^\star = \alpha m,\ p_2 x_2^\star = \beta m$, therefore the share of income spent on good 1 is α, and the share spent on good 2 is β.

4. Consider the same problem in question 3. Based on the demand functions you derive in question 3, first show that the demand function of each good has constant (cross) price elasticities over both prices, and determine whether good 1 is a substitute/complement/neither of good 2 by means of the cross price elasticity. Finally, calculate the income elasticity of demand for each good, and determine whether they are normal or inferior goods.

Solution: From the results in question 3, we have:

$\varepsilon_{11} = \frac{\partial x_1^\star}{\partial p_1} \cdot \frac{p_1}{x_1^\star} = -\frac{\alpha m}{p_1^2} \cdot \frac{p_1}{x_1^\star} = -\frac{\alpha m}{p_1 x_1^\star} = -\frac{\alpha m}{\alpha m} = -1$

$\varepsilon_{12} = \frac{\partial x_1^\star}{\partial p_2} \cdot \frac{p_2}{x_1^\star} = 0 \cdot \frac{p_2}{x_1^\star} = 0$

$\varepsilon_{21} = \frac{\partial x_2^\star}{\partial p_1} \cdot \frac{p_1}{x_2^\star} = 0 \cdot \frac{p_1}{x_1^\star} = 0$

$\varepsilon_{22} = \frac{\partial x_2^\star}{\partial p_2} \cdot \frac{p_2}{x_2^\star} = -\frac{\beta m}{p_2^2} \cdot \frac{p_2}{x_2^\star} = -\frac{\beta m}{p_2 x_2^\star} = -\frac{\beta m}{\beta m} = -1$

Therefore the demand function of each good shows constant (cross) price elasticities over both prices. Since the cross price elasticities $\varepsilon_{12} = \varepsilon_{21} = 0$, therefore the two goods are neither substitutes nor complements.

Income elasticities:

$\varepsilon_{1m} = \frac{\partial x_1^\star}{\partial m} \cdot \frac{m}{x_1^\star} = \frac{\alpha}{p_1} \cdot \frac{m}{x_1^\star} = \frac{\alpha m}{p_1 x_1^\star} = \frac{\alpha m}{\alpha m} = 1$

$\varepsilon_{2m} = \frac{\partial x_2^\star}{\partial m} \cdot \frac{m}{x_2^\star} = \frac{\beta}{p_2} \cdot \frac{m}{x_2^\star} = \frac{\beta m}{p_2 x_2^\star} = \frac{\beta m}{\beta m} = 1$

Since the income elasticities $\varepsilon_{1m} = \varepsilon_{2m} = 1 > 0$, therefore the two goods are both normal.

5. Consider a market with two consumers, 1 and 2, and two goods, 1 and 2. Assume that consumer 1's preference can be represented by a utility function: $U_1(x_1,\, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$, and consumer 2's preference can be represented by another utility function: $U_2(x_1,\, x_2) = x_1^{\frac{3}{4}} x_2^{\frac{1}{4}}$, where x_1 is the demand for good 1 and x_2 is the demand for good 2. Suppose that the price of good 1 is p_1, the price of good 2 is p_2, and that both consumers have the same income m. Assume that p_1, p_2, m are all positive.

Based on your results in question 3, first, for each consumer, write down his/her demand function for both goods (you should have two demand functions for each consumer), and then derive the market demand for each good. (Hint: recall the definition of market demand.)

Solution: Consumer 1's utility-maximizing problem is:

$$\underset{x_1, x_2 \geq 0}{Max}\; U_1(x_1,\, x_2) = x_1^{14} x_2^{34}$$
$$s.t.\; p_1 x_1 + p_2 x_2 \leq m$$

This is a special case of question 3: $\alpha = \frac{1}{4}$, $\beta = \frac{3}{4}$.

Therefore, according to the results you obtained in question 3, the demand functions for consumer 1 can be written as:

$$\begin{cases} x_{11}^{\star} = \frac{m}{4p_1} \\ x_{12}^{\star} = \frac{3m}{4p_2} \end{cases}$$

Similarly, consumer 2's utility-maximizing problem is also a special case of question 3: $\alpha = \frac{3}{4}$, $\beta = \frac{1}{4}$, and thus the demand functions for consumer 2 can be written as:

$$\begin{cases} x_{21}^{\star} = \frac{3m}{4p_1} \\ x_{22}^{\star} = \frac{m}{4p_2} \end{cases}$$

where $x_{ij}^{\star}$ denotes consumer i 's demand function of good j.

Market demand:At any price, the market demand is the sum of the amounts demanded by each of the consumers. So, if X_1 is the market demand for good 1, and X_2 is the market demand for good 2, then we have:

$$X_1 = x_{11}^{\star} + x_{21}^{\star} = \frac{m}{4p_1} + \frac{3m}{4p_1} = \frac{m}{p_1}$$

and

$$X_2 = x^\star_{12} + x^\star_{22} = \frac{3m}{4p_2} + \frac{m}{4p_2} = \frac{m}{p_2}$$

i.e. the market demand for each good is:

$$\begin{cases} X_1 = \frac{m}{p_1} \\ X_2 = \frac{m}{p_2} \end{cases}$$

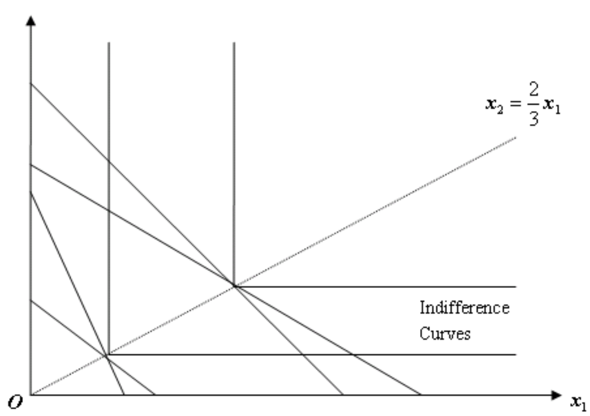
6. Consider an economy with a representative consumer and two goods, 1 and 2. Assume that the consumer's preference can be represented by a Leontief utility function: $U(x_1,\, x_2) = min(2x_1,\, 3x_2)$, where x_1 is the demand for good 1 and x_2 is the demand for good 2. Suppose that the price of good 1 is p_1, the price of good 2 is p_2, and that the consumer has an income m. Assume that p_1, p_2, m are all positive. Derive the demand function for each good.

Solution: From the question we can write the representative consumer's utility-maximizing problem as:

$$\underset{x_1, x_2 \geq 0}{Max}\ U(x_1,\, x_2) = min(2x_1,\, 3x_2)$$

$$s.t.\ p_1 x_1 + p_2 x_2 \leq m$$

Since the utility function has the Leontief form, then the two goods are perfect complements. Therefore the consumer will always choose the kink point where $2x_1 = 3x_2$, i.e. the maxima $(x^\star_1,\, x^\star_2)$ satisfies $2x^\star_1 = 3x^\star_2$. Why? We can use the indifference curves to see the fact.



Also since the consumer will spend all his/her income, we can have two variables $(x_1^\star,\, x_2^\star)$ and two equalities:

$$\begin{cases} 2x_1^\star = 3x_2^\star \\ p_1x_1^\star + p_2x_2^\star = m \end{cases}$$

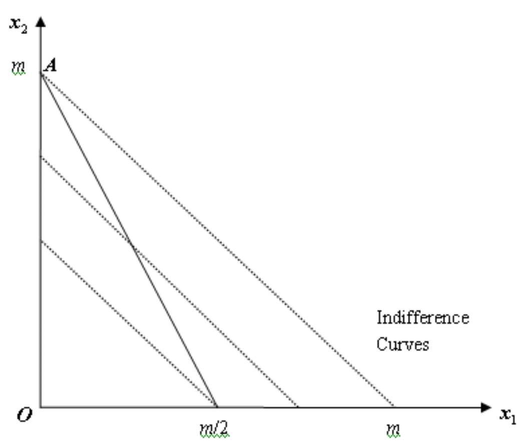
Therefore by solving the two equations, we can find the demand function for each good:

$$\begin{cases} x_1^\star = \frac{3m}{3p_1+2p_2} \\ x_2^\star = \frac{2m}{3p_1+2p_2} \end{cases}$$

7. Consider an economy with a representative consumer and two goods, 1 and 2. Assume that the consumer's preference can be represented by a linear utility function: $U(x_1,\, x_2) = ax_1 + bx_2$, where x_1 is the demand for good 1 and x_2 is the demand for good 2, a, b are both positive real numbers. Suppose that the price of good 1 is 2, the price of good 2 is 1, and that the consumer has a positive income m. For each of the following cases, draw both the budget line and the indifference curve associated with the linear utility function in a two-dimensional diagram with x_1 on the horizontal axis and x_2 on the vertical axis, and derive the demand function for each good. i). A special case where $a = b = 1$ ii). $a > 2b$ iii). $a < 2b$ iv). $a = 2b$

Solution:

i). A special case where $a = b = 1$



In the graph above, the dotted lines are the indifference curves for the utility function $U(x_1,\, x_2) = x_1 + x_2$, and the solid line is the budget line $2x_1 + x_2 = m$.

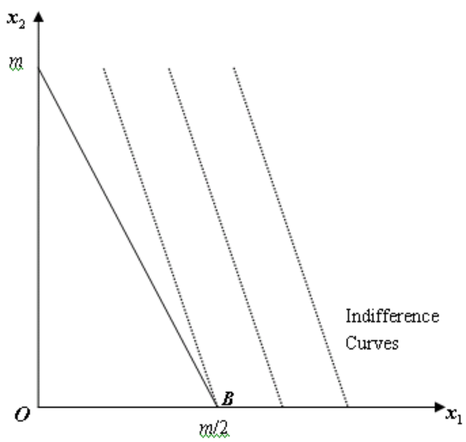
We can find that point A (or consumption bundle A) - the intersection of the budget line and the vertical axis - maximizes the utility under the budget constraint.

We can also calculate that the coordinate of point A is: $x_1^A = 0$, $x_2^A = m$.

Therefore the demand for good 1 is 0, and the demand for good 2 is m.

ii). $a > 2b$ In this case, in the two-dimensional diagram with x_1 on the horizontal axis and x_2 on the vertical axis, the slope of indifference curves is $-\frac{a}{b} < -2$, while the slope of the budget line is -2, meaning that the indifference curves are steeper than the budget line.

Therefore the graph is as follows:

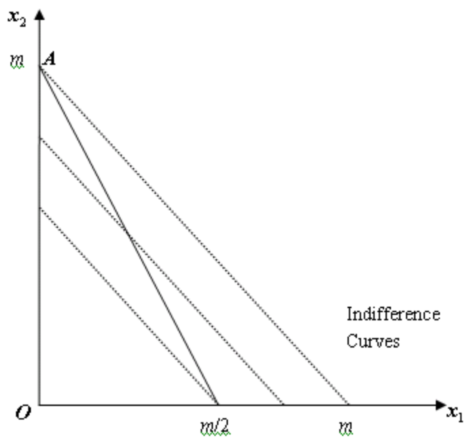


From the graph we can find that point B - the intersection of the budget line and the horizontal axis - maximizes the utility under the budget constraint. We can also calculate that the coordinate of point B is: $x_1^B = \frac{m}{2}$, $x_2^B = 0$.

Therefore the demand for good 1 is $\frac{m}{2}$, and the demand for good 2 is 0.

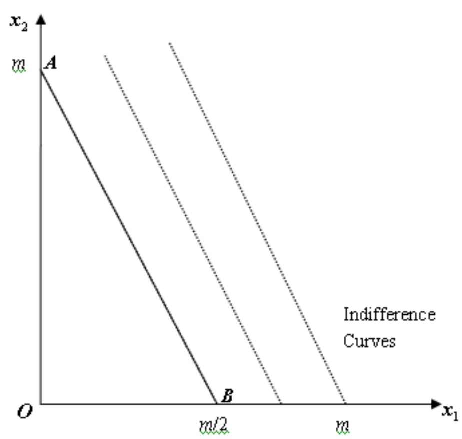
iii). $a < 2b$ In this case, the slope of indifference curves is $-\frac{a}{b} > -2$, while the slope of the budget line is -2, meaning that the indifference curves are flatter than the budget line.

Therefore the graph is as follows:



From the graph we can find that point A also maximizes the utility under the budget constraint in this case. Therefore the demand for good 1 is 0, and the demand for good 2 is m.

iv). $a = 2b$ In this case, the slope of indifference curves is $-\frac{a}{b} = -2$, meaning that the indifference curves have the same slope as the budget line. Therefore the graph is as follows:



From the graph we can find that all the points on the budget line give the same utility level and therefore any point (or consumption bundle) maximizes the utility under the budget constraint.

Therefore if we write the demands for good 1 and 2 as $x_1^\star$ and $x_2^\star$ respectively, then $x_1^\star$ and $x_2^\star$ satisfy these conditions:

$$2x_1^\star + x_2^\star = m,\ x_1^\star \geq 0, \text{ and } x_2^\star \geq 0$$

In other words, any point (or consumption bundle) $(x_1^\star, x_2^\star)$ that satisfy the above conditions can be the demand for both goods.